Numerieke methoden in de natuurwetenschappen

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Ex. 5: Solution of initial value differential equations

In this exercise you will solve the same problem with three different algorithms. In the final report you should comment about the advantages/disadvantages of each method.

Consider a sphere falling from rest in presence of air resistance. The rate of change of velocity is given by

$$m\frac{dv}{dt} = mg - kv^2 \tag{1}$$

where $m = 10^{-2}$ kg is the mass of the object, v the velocity, $g = 9,8$ ms^{-2} is the acceleration due to gravity and $k = 10^{-4}$ kg/m is the drag coefficient. You know what to expect, the sphere will be first accelerated by gravity but with increasing velocity the friction term will eventually lead to a motion with constant velocity v_{lim}. This simple problem has been chosen for the following reasons:

- it has an analytical solution for $v(t)$ to check the numerical error.
- there are different regimes of the motion (linear, rapidly varying, constant) and it will be clear that algorithms with adjustable time step are particularly suited in such situations.
- one can calculate both velocity and position of the falling body by reducing the second order equation of motion to a system of two coupled first order differential equations.

In writing the program, keep all subroutines for integration in a separate module which does use any other module. All quantities needed are passed as arguments of the call and as external functions. In this way, you can directly use these subroutines also for other problems in the future. Make also use of an input file to read initial conditions, values of friction coefficient, time range of integration, number of equations to be integrated and whatever constitutes an input variable.

a) Use the Euler and Runge Kutta method of order 2 and 4 to calculate the velocity for $0 < t < 10$ s distributed over a uniform grid with spacing δt. In order to find an appropriate value for the time step δt check the results against the expected behaviour and value of v_{lim}.

As already mentioned the behaviour of the sphere goes through different regimes and integration over a uniform grid in time is obviously not the most efficient choice. The

purpose of the following two exercises is to apply other algorithms where the time step is chosen at every step in order to keep the precision constant.

b) Use the Runge Kutta method with adaptive step size (step doubling) to find the velocity of the sphere for $0 < t < 10$ s with tolerance 10^{-3} and 10^{-4}. Compare the solutions and the steps used in the calculation. Generalize to a system of two coupled equations to calculate velocity and position as a function of time.

c) Calculate the solution also by the Runge Kutta Fehlberg method (also called embedded Runge Kutta method) as in point b). The method is described in Numerical Recipes in Fortran, ch.16.2. An on-line version of this book can be found in our website Telescience.