Numerieke methoden in de natuurwetenschappen

Ex. 6: Systems of Ordinary Differential Equations; the anharmonic oscillator
Consider the one dimensional motion of a particle subjected to a restoring force described by a potential of the form:

$$V(x) = A\frac{|x|^{B+1}}{(B+1)} \tag{1}$$

so that the restoring force is

$$K(x) = -A|x|^B \frac{x}{|x|} \tag{2}$$

The equation of motion is:

$$-M\frac{d^2x}{dt^2} - A|x|^B \frac{x}{|x|} = 0 \tag{3}$$

Take M=1kg, A=1Nm^{-1}. The initial conditions at $t = 0$ are x=1m, and dx/dt=0.

Reduce the equation to a system of two first-order differential equations. Consider first the harmonic case $B = 1$. Calculate the period of vibration and compare it to the analytical value of $2\pi\sqrt{M/A}$. Identify the method more suitable to obtain conservation of total (potential+kinetic) energy over a few periods of vibration. Explain your considerations in achieving this result.

Next consider other two values of B of your choice and compare displacements and velocity with the harmonic case.