Note Title 28/11/2007

Application to polymers

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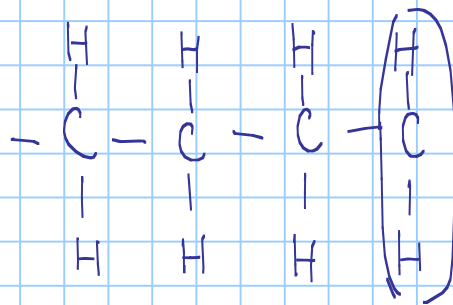
22 Nov 2007



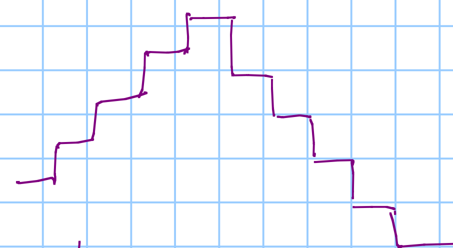
$-A-A-A-A-$

polymer $N \gg 1$

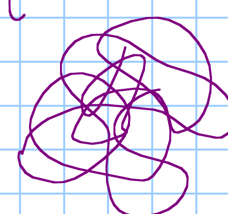
monomer

Continuum medium approximation

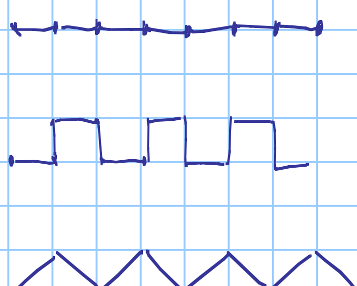
Usually polymers are solids



Single chain in a good solvent



$T=0$

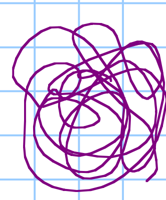


$S = \ln W$

$W = 1$ unique structure

$S = 0$

$\underline{F = E - TS}$



globe with size R

N monomers : $R \sim N^{\nu}$

$\nu > 0$

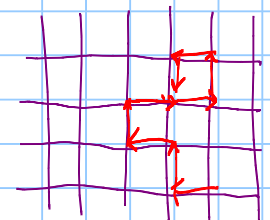
aim to calculate

(1) rigid chain $T=0$ $\nu = 1$

(2) ideal chain : no interactions between monomers

$\nu = 1/2$

Ideal chain (d-dimensional space)



$$\vec{R} = \sum_{j=1}^{N} \vec{\rho}_j \qquad |\vec{\rho}_j| = a$$

$$\vec{\rho}_j = \pm(a,0,0) = \pm(0,a,0) = \pm(0,0,a)$$

$$\langle \vec{\rho}_i \vec{\rho}_j \rangle = 0 \quad i \neq j \qquad \langle \vec{\rho}_j \rangle = 0 \qquad \langle \vec{R} \rangle = 0$$

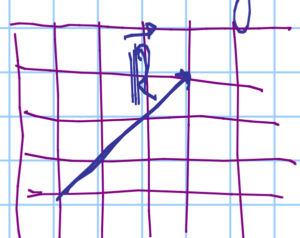
$$\langle \vec{R}^2 \rangle = \sum_{j=1}^{N} \langle \vec{\rho}_j \vec{\rho}_j \rangle = N a^2 \qquad i=j \quad \langle \vec{\rho}_j \vec{\rho}_j \rangle = a^2$$

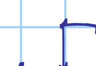
$$\boxed{R \simeq \sqrt{\langle \vec{R}^2 \rangle} \sim \sqrt{N}}$$

The same story as for Brownian motion when R plays role of time $\langle R^2 \rangle \sim t$

no memory [model for a drunk man coming back home]

Pure idealisation. Usually molecules know the angle of bonding.



$\eta_N(\vec{R})$ number of ways  from 0 to $\vec{R}$ for N steps

$$\sum_{\vec{R}} \eta_N(\vec{R}) = z^N \qquad \begin{array}{ll} z = 4 & d = 2 \\ z = 6 & d = 3 \end{array}$$

probability $\quad P_N(\vec{R}) \sim \left\langle \delta\left(\vec{R} - \sum_{j=1}^{N} \rho_j\right) \right\rangle$

$$\delta\left(\vec{R}-\sum_{j=1}^{N} p_j\right) = \int \frac{d^d k}{(2\pi)^d}\, e^{i\vec{k}\vec{R}} \cdot e^{-i\vec{k}\sum_{j=1}^{N} p_j}$$

since all steps are independent

$$\left\langle e^{-i\vec{k}\sum_{j=1}^{N} p_j} \right\rangle = \prod_{j=1}^{N} \left\langle e^{-i\vec{k}\vec{p}_j}\right\rangle = \left\langle e^{-i\vec{k}\vec{p}}\right\rangle^{N}$$

$N \to \infty$
$a \to 0$
$N a^2 = \text{const}$

$$\left\langle e^{-i\vec{k}\vec{p}} \right\rangle = 1 - i\vec{k}\langle\vec{p}\rangle - \frac{1}{2}\left\langle (\vec{k}\vec{p})^2 \right\rangle$$

$$\langle\vec{p}\rangle = a \qquad \left\langle (\vec{k}\vec{p})^2 \right\rangle = \frac{1}{d} k^2 \langle\vec{p}\rangle^2 = \frac{1}{d} k^2 \frac{\langle\vec{R}^2\rangle}{N}$$

$$\left\langle e^{-i\vec{k}\sum_{j=1}^{N}\vec{p}_j} \right\rangle \simeq \left(1 - \frac{k^2}{2dN}\langle\vec{R}^2\rangle\right)^N \simeq e^{-\frac{k^2\langle R^2\rangle}{2d}}$$

$$P_N(\vec{R}) \sim \int \frac{d^d k}{(2\pi)^d}\, e^{i\vec{k}\vec{R}} \exp\left(-\frac{k^2\langle R^2\rangle}{2d}\right) \sim \exp\left(-\frac{R^2 d}{2\langle R^2\rangle}\right)$$

Ideal chain

$$F(R) = E - TS = \text{const} + \text{const}' \frac{R^2}{N}$$

$$E = 0$$

$$P(R) \sim e^{-F(R)} \sim \exp\left(-\frac{R^2}{N}\right) \qquad \langle R^2\rangle \sim N$$

Real chain : mean-field (kind) for polymers

Flori (Nobel Prize)

$$\bar V(r) = \begin{cases} \infty, & r \le a \\ 0, & r > 0 \end{cases}$$

$$E_{int}(R) \sim v c^2 R^d \sim \frac{v N^2}{R^d}$$

$$F(R) = c_1 \frac{R^2}{N} + c_2 \frac{N^2}{R^d}$$ ← assumption brilliant idea

$$\frac{d F(R)}{dr} = 0 \qquad \frac{d^2 F}{dR^2} > 0 \qquad R^{2+d} \simeq N^3$$

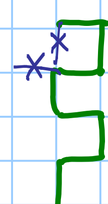
$$R \sim N^{3/(2+d)}$$

$$\boxed{\nu = \frac{3}{d+2}}$$

$d = 3 \quad \nu = 0.6$

↑ good agreement with experiment

des Cloizeaux
de Gennes



Random walks with avoided self-crossing

Sometimes you have 3 directions or 2 or 1

$Q_N(ij)$ — number of ways of the length N $i \to j$ without self-crossing

$$\boxed{Q_N^{tot} \sim \tilde{z}^N N^{\gamma-1}} \qquad \eta_N^{tot} = z^N$$

$z = 6 \qquad \tilde{z} < z \qquad \tilde{z} = 4.68$

$\tilde{z}$ is not universal $\qquad \gamma$ is universal

$\gamma = 7/6 \quad 3D \qquad \gamma = 4/3 \quad 2D$

$\sim . \quad \sim . \quad \sim . \quad \sim . \quad \sim . \quad \sim .$

$$S_i^\alpha \qquad \mathcal{H} = -K \sum_{\langle ij \rangle} S_i^\alpha S_j^\alpha , \qquad \alpha = 1, 2, \dots n$$

$n = 1$ Ising model $\qquad \sum_{\alpha=1}^{n} S_{i\alpha}^2 = n$

If you calculate corr. func. for this model
$n = 0$ describes polymers random walks with...

$$Z = \sum_{\{S_i\}} e^{-\mathcal{H}} = \sum_{\{S_i\}} \left(1 + K \sum_{\langle ij \rangle \alpha} S_i^\alpha S_j^\alpha + \frac{K^2}{2} \left(\sum_{\langle ij \rangle \alpha} S_i^\alpha S_j^\alpha \right)^2 + \dots \right)$$

High T avaraging $\langle \dots \rangle_0 \equiv$ average with $\mathcal{H} = 0$

$$\langle S_i^\alpha \rangle_0 = 0 \qquad \langle (S_i^\alpha)^2 \rangle_0 = 1 \qquad \langle S_i^\alpha S_j^\beta \rangle = 0, \ i \neq j$$

$$f(K) = \left\langle e^{-i \sum_\alpha K_\alpha S_i^\alpha} \right\rangle_0 = 1 - i \sum_\alpha K_\alpha \langle S_i^\alpha \rangle_0 + \frac{1}{2} \sum_{\alpha\beta} K_\alpha K_\beta \langle S_i^\alpha S_i^\beta \rangle_0$$

$$\frac{\partial^2 f}{\partial K_\alpha^2} = - \left\langle (S_i^\alpha)^2 e^{i \sum_\alpha K_\alpha S_i^\alpha} \right\rangle_0$$

$$\sum_{\alpha} \frac{\partial^2 f}{\partial K_\alpha^2} = -n f(K) \qquad f = f(|\vec{K}|)$$

direction independent

$$|\vec{K}| = \left(\sum_{\alpha=1}^{n} K_\alpha^2 \right)^{1/2} \qquad \frac{\partial f}{\partial K_\alpha} = \frac{\partial f}{\partial K} \frac{\partial K}{\partial K_\alpha}$$

$$\frac{\partial^2 f}{\partial K_\alpha^2} = \frac{1}{K}\frac{\partial f}{\partial K} + \frac{K_\alpha^2}{K}\frac{\partial}{\partial K}\left(\frac{1}{K}\frac{\partial f}{\partial K}\right)$$

$$\sum_{\alpha} \frac{\partial^2 f}{\partial K_\alpha^2} = \frac{n}{K}\frac{\partial f}{\partial K} + K\frac{\partial}{\partial K}\left(\frac{1}{K}\frac{\partial f}{\partial K}\right) = \frac{\partial^2 f}{\partial K^2} + \frac{n-1}{K}\frac{\partial f}{\partial K} = -nf$$

$$\frac{\partial^2 f}{\partial K^2} + \frac{n-1}{K}\frac{\partial f}{\partial K} + nf = 0$$

$$f(K=0) = 1 \qquad n=0 \qquad f(K) = 1 - \frac{1}{2}K^2$$

$$\frac{\partial f}{\partial K}(K=0) = 0$$

Taylor expansion for $n=0$ stops at II order

$$\langle S_i^{\alpha_1} S_i^{\alpha_2} S_i^{\alpha_3} S_i^{\alpha_4} \rangle_0 = 0 \qquad \langle S_i^{\alpha} S_i^{\beta} \rangle = \delta_{\alpha\beta}$$

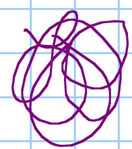
$$Z = \left\langle \prod_{ij} \left(1 + K\sum_{\alpha} S_i^\alpha S_j^\alpha + \frac{K^2}{2}\sum_{\alpha}(S_i^\alpha S_j^\alpha)^2 + \dots \right)\right\rangle_0 = 1$$

$$\sum_{\alpha} \langle S_i^\alpha S_i^\alpha \rangle = n \xrightarrow[n\to 0]{} 0 \qquad G_\alpha(i,j) = \langle S_i^\alpha S_j^\alpha e^{-\mathcal{H}} \rangle_0$$

$$\langle S_i^\alpha S_j^\alpha \rangle_{n=0} = \sum_{N} Q_N(i,j) K^N$$

29 Nov 2007

$R \sim N$



ideal $R \sim N^{0.5}$

$R \sim N^{0.6}$ (Flori)

$\mathcal{H} = K \sum_{\langle ij \rangle \alpha} S_i^\alpha S_j^\alpha$ $\alpha = 1, 2 \dots n$

$\langle ij \rangle$ - neighbouring atoms

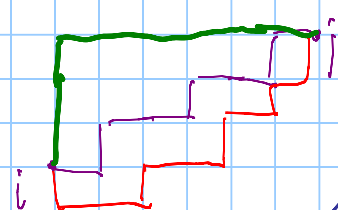
$\sum_{\alpha=1}^{n} S_\alpha^2 = n$ $\langle S_i^\alpha S_i^\beta \rangle = \delta_{\alpha\beta}$

$n = 0$ number of order parameters

Calculate corr. func. of spins

$$\langle S_i^\alpha S_j^\beta \rangle_{n=0} = \sum_N K^N Q_N(i, j)$$

all pathes without self-crossing



$\langle S_i^\alpha S_i^\alpha \rangle = 1$

$\langle S_i^{\alpha_1} S_i^{\alpha_2} S_i^{\alpha_3} S_i^{\alpha_4} \rangle = 0$

We still do not have critical point it just a corr. func. We need to establish

$Q_N^{tot} = \text{const}\ \bar{z}^N N^{\gamma-1}$ result of computer simul

$\eta_N^{tot} = z^N$, $z = 6$ $\bar{z} < z$ $\bar{z} \approx 4.68$ sc lattice

$$\chi = \sum_{ij} \langle S_i^\lambda S_j^\lambda \rangle = \sum_N K^N Q_N^{tot} \sim \sum_N (K\bar{z})^N N^{\gamma-1}$$

susceptibility $\qquad K\bar{z} = 1$ critical point

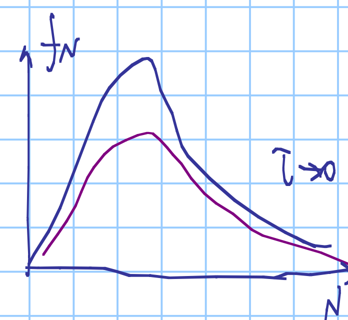
if $K\bar{z} < 1$ series converge

$K\bar{z} > 1$ series diverge

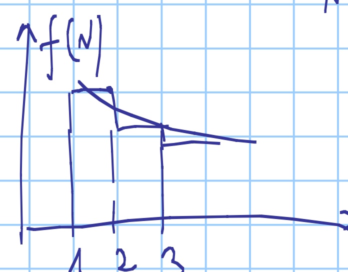
K was arbitrary $\qquad K\bar{z} \to 1 \qquad K\bar{z} < 1$

$$K\bar{z} \simeq e^{-\tau}, \quad \tau \to 0, \quad \tau > 0$$

$$\chi \sim \sum_N e^{-\tau N} N^{\gamma-1}, \quad \tau \to +0$$



$$\sum_N f(N) \simeq \int_0^N dN\, f(N)$$



$$\chi \sim \int_0^\infty dN\, e^{-\tau N} N^{\gamma-1} \sim \tau^{-\gamma}$$

but this is not very interesting

$$\langle S_i^\lambda S_j^\lambda \rangle_{n=0} = \sum_N K^N Q_N^{tot} \frac{Q_N(i,j)}{Q_N^{tot}} \sim \sum_N e^{-\tau N} N^{\gamma-1} \ldots$$

$$\ldots \frac{Q_N(i,j)}{Q_N^{tot}} \sim \int_0^\infty dN\, N^{\gamma-1} e^{-\tau N} \frac{Q_N(i,j)}{Q_N^{tot}}$$

$$\frac{Q_N(i,j)}{Q_N^{tot}} \simeq N^{-\nu} f(R_{ij} N^{-\nu})$$

The idea of scaling ... close to crit point there is only one natural length scale

$R \sim N^{-\nu}$  radius of globe

$$\langle S_i^\alpha S_j^\alpha \rangle \sim \int_0^\infty dN\, N^{\gamma-1-\nu} e^{-\tau N} f(R_{ij} N^{-\nu}) =$$

$$= \left[N = \frac{t}{\tau} \right] \sim \frac{1}{\tau^{\gamma-\nu}} \int_0^\infty dt\, t^{\gamma-1-\nu} e^{-t} f(R_{ij} t^{-\nu} \tau^{\nu})$$

we assumed scaling for polymers we did not prove it

$$\langle S_i^\alpha S_j^\alpha \rangle = \frac{1}{R^{d-2+\eta}} f(R_{ij} \tau^{-\nu}) \quad \ldots \quad \nu \text{ is real?}$$

$d = 4 - \epsilon$

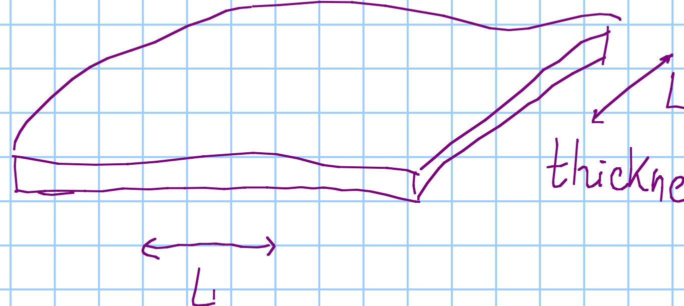
$$\nu(n) = \frac{1}{2} + \frac{n+2}{4(n+8)} \epsilon + \frac{(n+2)(n^2+23n+60)}{8(n+8)^2} \epsilon^2 + \ldots$$

$\epsilon = 1 \qquad n = 0 \qquad \nu = 0.59 \simeq 0.6$

Wilson theory works.

Statistical Physics of Membranes

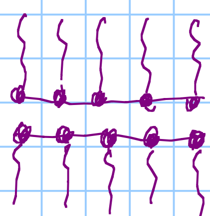
D. Nelson, T. Piran, S. Weinberg
"Statistical Mechanics of Membranes and Surfaces"
(World Scientific 2004)



thickness $\to 0$

amphiphilic molecules

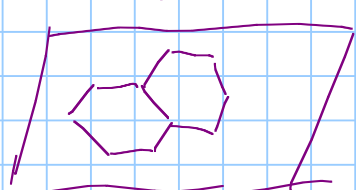
hydrophobic
hydrophilic



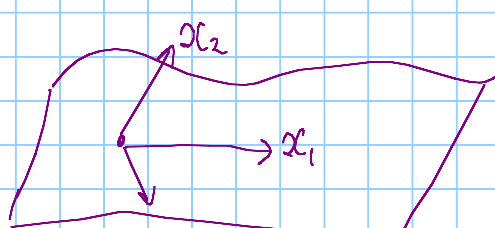
membrane
living cells

bolaamphiphilic

graphene discovered exp-ly 3 years ago
single atomic layer of C

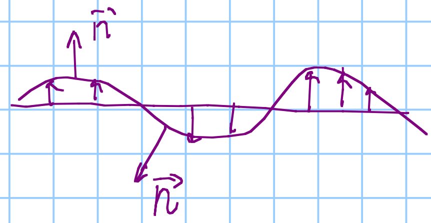


the thinnest possible membrane



$$z = (x_1, x_2, f(x_1, x_2))$$

$f = f(x_1, x_2)$ height

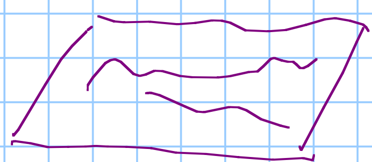


$\vec{n}$ - is normal to membrane surface

$|\vec{n}| = 1$

$$\vec{n}(x_1, x_2) = \frac{\left(-\frac{\partial f}{\partial x_1}, -\frac{\partial f}{\partial x_2}, 1\right)}{\sqrt{1 + |\nabla f|^2}} \qquad |\nabla f|^2 = \left(\frac{\partial f}{\partial x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2}\right)^2$$

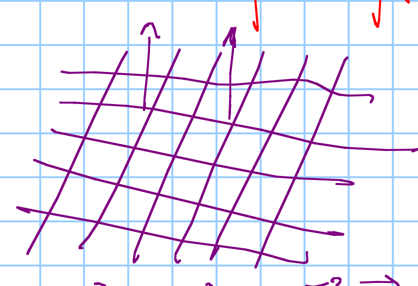
$T = 0 \qquad f(x_1, x_2) = 0 \qquad$ flat



Bending energy
Bending rigidity

$$\mathcal{H} = -\varkappa \sum_{\langle ij \rangle} \vec{n}_i \vec{n}_j$$

deviation from flatness



$\vec{n}(x_1, x_2)$ – smooth function

$\vec{n}_i \parallel \vec{n}_j \qquad \vec{n}_i \vec{n}_j = \pm 1$

$$\vec{n}(x_1, x_2) \simeq \vec{n}_0 + \delta\vec{n}$$

$\vec{n}_0 = (0, 0, 1) \qquad \delta\vec{n} \perp \vec{n}_0$

$$\delta\vec{n} = \left(-\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 0\right) \qquad |\nabla f|^2 \ll 1$$

if I rotate my plane as whole it does not require the energy.

$$\boxed{\mathcal{H} = \frac{1}{2}\varkappa \int d^2x \left[\left(\frac{\partial n}{\partial x_1}\right)^2 + \left(\frac{\partial n}{\partial x_2}\right)^2\right]}$$

bending energy

Up to second order $\delta\vec{n} = \left(-\frac{\partial f}{\partial x_1}, -\frac{\partial f}{\partial x_2}, -\frac{1}{2}(\nabla f)^2\right)$

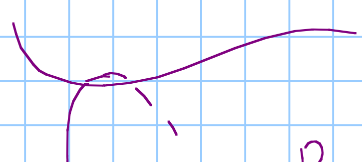
$$\mathcal{H} \simeq \frac{\varkappa}{2}\int d^2x \left[(\nabla^2 f)^2 - 2\det\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)\right] + O(f^3)$$

$\det\left(\frac{\partial^2 f}{\partial x_1 \partial x_2}\right) \sim$ total derivative

$$2\det\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right) = -\sum_{nm} \epsilon_{in}\epsilon_{jm} \frac{\partial^2}{\partial x_n \partial x_m}\left[\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}\right]$$

$$\boxed{\mathcal{H} = \frac{\varkappa}{2}\int d^2x \, (\nabla^2 f)^2}$$

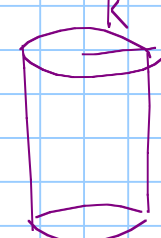
$\epsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$



R_1, R_2

$R_1 > 0 \quad R_2 < 0$

saddle



R

$R_1 = R$

$R_2 = \infty$

$K = (R_1 R_2)^{-1}$

Gaussian curvature

$$\boxed{\int dS \, K = 4\pi(1-g)}$$

Gauss-Bonnet theorem

g – genus

$g=0$ – sphere

$g=1$ – torus

This det is nothing but Gaussian curvature

$\Rightarrow$ $\int dS\, K = \text{const}$ responsible for the change of topology, which we do not consider, since we consider only small deformation. But some fluctuations can change topology.

Height fluctuations

$$Z = \int \mathcal{D}f\, e^{-\mathcal{H}/T} = \int \mathcal{D}f \exp\left[-\frac{\varkappa}{2T}\int d^2x\,(\nabla^2 f)^2\right]$$

$$f = \sum_q f_q e^{i\vec{q}\vec{x}} \qquad\qquad \sum_q = \frac{L^2}{(2\pi)^2}\int d^2 q$$

$$Z \sim \int \prod_q df_q \exp\left(-\frac{\varkappa}{2T}\sum_q |q^2 f_q|^2\right)$$

$$\int d^2x\,(\nabla^2 f)^2 = \sum_q q^4\, |f_q|^2 \qquad\qquad \langle |f_q|^2\rangle = \frac{T}{\varkappa q^4}$$

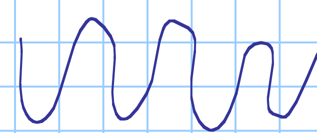
Membrane is fluctuating what is the typical height

$$\langle f^2\rangle = \int \frac{d^2q}{(2\pi)^2}\langle |f_q|^2\rangle = \int_0^\infty \frac{q\,dq}{2\pi}\,\frac{T}{\varkappa q^4} =$$

$$= \frac{T}{2\pi\varkappa}\int_0^\infty \frac{dq}{q^3} \ldots$$ this integral does not exist

$q_{min} \simeq \frac{1}{L}$ $\qquad$ $\langle f^2 \rangle = \frac{T}{2\pi\varkappa}\int_{q_{min}}^{\infty}\frac{dq}{q^3} \sim \frac{T}{2\pi\varkappa}\frac{1}{q_{min}^2} \sim L^2$

typical height fluctuations $\sim L$ for large samples is infinite. $\Rightarrow$ flat membr. cannot exist

they would be crumpled 

$$\langle |n_q|^2 \rangle \simeq q^2 \langle |f_q|^2 \rangle \simeq \frac{T}{\varkappa}\frac{1}{q^2}$$

$$\langle \vec{n}(0)\,\vec{n}(\vec{R}) \rangle = \int \frac{d^2q}{(2\pi)^2} \langle |\vec{n}_q|^2 \rangle \cos \vec{q}\vec{R}$$

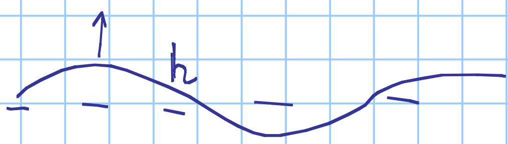
$$\langle \vec{n}(0)\,\vec{n}(R) \rangle \sim \frac{T}{\varkappa} \ln\left(\frac{L}{a}\right) \qquad \int \frac{d^2q}{q^2} \sim \ln q_{min}$$

But flat membrane do exist (graphene) $\Rightarrow$

predicts crumpled membrane. This theory is like Ornstein-Zernike and it is not enough to describe... Expression for bending energy should be updated.

⋮

6 Dec 2007



$x_1, x_2, x_3 \quad h(x_1, x_2)$

$\vec{n} \sim -\nabla h$

$$\mathcal{H} = \frac{\varkappa}{2} \int d^2x \, (\nabla^2 h)^2$$

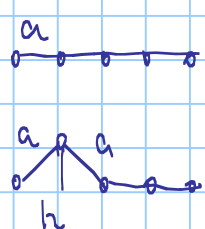
$$\langle |h_{\vec{q}}|^2 \rangle = \frac{T}{\varkappa q^4}$$

$$\langle h^2 \rangle = \sum_q{}' \langle |h_q|^2 \rangle \sim \frac{T}{\varkappa} L^2$$

$q_{min} = \frac{1}{L}$ to have physical picture

$$\langle n(\vec{r})\, n(0) \rangle \sim \frac{T}{\varkappa} \ln \frac{r}{a}$$

Prediction that all membranes should be crumpled



$\delta a \sim \frac{h^2}{a}$

$h \ll a$

we create deviation of atoms in plane which was not considered in $\mathcal{H}$.

$$\vec{x} \to \vec{x} + \vec{u}(\vec{x}) \qquad dl^2 = \sum_{i=1}^{3} dx_i^2 \longrightarrow \sum_{i=1}^{3} (dx_i + du_i)^2$$

$$du_i = \sum_j \frac{\partial u_i}{\partial x_j} dx_j \qquad dl'^2 = dl^2 + 2\sum_{ij} \bar{u}_{ij}$$

$$u_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \sum_k \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right)$$

deformation tensor

$$\mathcal{H}_{elastic} = \frac{1}{2}\int d^d x \left[2\mu \sum_{ij} \bar{u}_{ij}^2 + \lambda \left(\sum_{k} \bar{u}_{kk} \right)^2 \right]$$

$$\sum_{ij} \bar{u}_{ij}^2 = \mathrm{Tr}\,\hat{\bar{u}}^2 \qquad \left(\sum_k \bar{u}_{kk} \right)^2 = (\mathrm{Tr}\,\hat{u})^2$$

$\bar{u}_{ij} = 0$ minimum

$$\hat{A}' = \hat{S}\hat{A}\hat{S}^{-1}$$

$$\mathrm{Tr}\,\hat{A}' = \mathrm{Tr}\,\hat{A}$$

μ, λ – elastic constants
Lame coeff

$$\mathcal{H} = \frac{1}{2}\int d^d x \left[\varkappa (\nabla^2 h)^2 + 2\mu \sum_{ij} \bar{u}_{ij}^2 + \lambda \left(\sum_{k=1,2} \bar{u}_{kk} \right)^2 \right]$$

$$\bar{u}_{ij} \approx \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial h}{\partial x_i}\frac{\partial h}{\partial x_j} \right)$$

skipped $\dfrac{\partial u_k}{\partial x_i}\dfrac{\partial u_k}{\partial x_j}$ $\qquad \left| \dfrac{\partial u_i}{\partial x_j} \right| \ll 1$

fluctuations out of plane are big, but within the plane are small

$$Z = \int \mathcal{D}\vec{u}\, \mathcal{D}h\, e^{-H/T} \qquad \bar{u} = (u_1, u_2)$$

$$\langle \vec{n}(\vec{r})\, \vec{n}(0) \rangle = \int \mathcal{D}\vec{u}\, \mathcal{D}h\, e^{-H/T}\, \vec{n}(\vec{r})\, \vec{n}(0)$$

$$\mathcal{H}_{el} = \frac{\varkappa}{2}\int d^2 x\, (\nabla^2 h)^2 + H_{int}$$

$$H_{int} = \frac{1}{2}\sum_{q_1, q_2, q_3} \vec{R}(\vec{q}_1, \vec{q}_2, \vec{q}_3)\, h_{\vec{q}_1} h_{q_2} h_{q_3} h_{-q_1} h_{-q_2} h_{-\vec{q}_3}$$

fourth-order term, but it is the same problem as Landau-Ginzburg hamiltonian

$$\mathcal{H} = \frac{1}{2}\int d^d x \left[r\varphi^2 + c(\nabla\varphi)^2 \right] + u\int d^d x\, \varphi^4$$

you can try to use perturbation theory
There is an effective renormalisation of

$$\varkappa \longrightarrow \varkappa_R(\vec{q})$$

$$\varkappa_R(\vec{q}) \simeq \varkappa + TK_0 \int \frac{d^2 k}{(2\pi)^2} \frac{\left[1 - \frac{|\vec{q}\,\vec{k}|^2}{K^2}\right]^2}{\varkappa\,|\vec{q}+\vec{k}|^4}$$

$$K_0 = \frac{4\mu(\mu+\lambda)}{2\mu+\lambda}$$ rescaling bending rigidity

$$\delta\varkappa(q) \simeq \frac{3TK_0}{8\pi\varkappa q^2} \xrightarrow[q\to 0]{} \infty$$ for long length fluctuations

the theory becomes unapplicable, because corrections to fluctuations $\gg$ main prediction

"Ginzburg criterion" q^* $\delta\varkappa(q^*) = \varkappa$

$$q^* = \frac{1}{\varkappa}\sqrt{\frac{3TK_0}{8\pi}}$$ $q < q^*$ simple theory is wrong

Temperature here is not important important dimensionality! In 1D you have fluctuatioeven $T=0$! The smaller dimensionality the higher fluctuations $\Rightarrow$ dangerous.

Scaling in q!

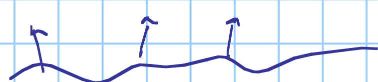
$$\varkappa \to \varkappa_R(q) \sim q^{-\eta_\varkappa}$$

$$\mu_q \sim \lambda_q \sim q^{\eta_u}$$

typical height of fluctuations $\bar{h} = \langle h^2 \rangle^{1/2} \sim L^{\zeta}$, $\zeta < 1$

membrane is more or less flat

$$\langle \vec{n}(\vec{r})\,\vec{n}(0)\rangle \xrightarrow[r\to\infty]{} \text{const}$$



$q < q^*$

$$\eta_u + 2\eta_\varkappa = 2$$

$$\zeta = 1 - \frac{\eta_\varkappa}{2}$$

You have 2D membrane in 3D

$D = 2$ $\quad d = 3$ $\quad \frac{1}{d-D}$

leading order in ε-expansion $\quad \eta_\varkappa = 1$

Real result: numerical methods $\eta_\varkappa \cong 0.8$

$\zeta \cong 0.6$ $\quad \bar{h} \sim L^{0.6}$

Graphene has ripples. The theory also predicts a negative Poisson ration (contrintuitive