Kubo formula

Linear response: electric current as a response to electric field applied

use the following representation $\vec{E}(t) = -\frac{\partial \vec{A}}{\partial t}$!

electric field

use $\vec{A} = \frac{\vec{E}_0}{i\Omega} e^{-i\Omega t}$, Ω - frequency,

time-independent electric field corresponds to the limit $\Omega \to 0$

Our task is to compute $\langle j(r,t) \rangle = -\frac{i}{V} \mathrm{Tr}_r (\hat{j}\, G^<)$ to the linear order in $\vec{E}$

average over space for convenience

The computation works out as this:

$$\langle j \rangle = -i \frac{1}{V} \mathrm{Tr}_r \left(\hat{j}_r \underbrace{\delta G^<}_{\text{first order in } A} \right) + i \frac{1}{V} \mathrm{Tr}_r \left(\frac{e^2}{m} A \right) \underbrace{G^{<(0)}}_{\text{equilibrium}}$$

$$j_r = \frac{-ie}{2m} \left(\vec{\nabla} - \overleftarrow{\nabla} \right)$$

$G^<$ is taken at coinciding arguments

$$G^< = G^<(r\,t; r, t)$$

We would like to use Wigner transform in $\delta G^<$

$\Rightarrow \quad \delta G^<(\varepsilon, T; r_1, r_2)$ (only in time)

$$\Rightarrow \quad \langle j \rangle_1 = -\frac{i}{V} \mathrm{Tr}_r\, \hat{j}_r\, \delta G^< = -\frac{i}{V} \underbrace{\int \frac{d\varepsilon}{2\pi} \mathrm{Tr}_r \left(j_r\, \delta G^<(\varepsilon, t_i) \right)}_{}$$

part of the trace over time is written explicitely.

Remember Dyson equation for $\hat{G} = \begin{pmatrix} G^R & G^K \\ 0 & G^A \end{pmatrix}$

as a response to the perturbation potential $\underbrace{-j_r \cdot \bar{A}(t)}_{} = V(r,t)$

Again write convolutions in time explicitely:

$$\delta \hat{G}(1,2) = \int_{-\infty}^{+\infty} dt_3\; \hat{G}^{(0)}(t_1, t_3)\; jA(t_3)\; \hat{G}^{(0)}(t_3, t_2)$$

Calculate Wigner transform for δG: (I omit (0) for compactness)

$$\delta G(\varepsilon, T) = \int d\underbrace{(t_1 - t_2)}_{\substack{\parallel \\ t}} \overbrace{}^{e^{i\varepsilon(t_1-t_2)}} \iint \frac{d\varepsilon_1}{2\pi} e^{-i\varepsilon_1(t_1 - t_3)}\, G(\varepsilon_1)\; jE_0 \frac{e^{-i\Omega t_3}}{i\Omega} \int \frac{d\varepsilon_2}{2\pi} e^{-i\varepsilon_2(t_3 - t_2)}\, G(\varepsilon_2)\, dt_3$$

does not depend on $\frac{t_1 + t_3}{2}$ since it is equilibrium use $t = t_1 - t_2$, $T = \dfrac{t_1 + t_2}{2}$ $\Rightarrow$ $t_1 = T + \dfrac{t}{2}$, $t_2 = T - \dfrac{t}{2}$

$$\delta G = \underbrace{\int dt \int dt_3}_{\text{integrate over times first}} \int \frac{d\varepsilon_1}{2\pi} \int \frac{d\varepsilon_2}{2\pi}\, e^{i\varepsilon t}\, e^{-i\varepsilon_1 (T + \frac{t}{2} - t_3)}\, e^{-i\Omega t_3}\, e^{-i\varepsilon_2 (t_3 - T + \frac{t}{2})}\, \frac{1}{i\Omega} G(\varepsilon_1)\, \bar{j}\bar{E}_0\, G(\varepsilon_2)$$

$$\Rightarrow \; 2\pi\, \delta\left(\varepsilon - \frac{\varepsilon_1}{2} - \frac{\varepsilon_2}{2}\right) \cdot 2\pi\, \delta(\varepsilon_1 - \varepsilon_2 - \Omega)$$

use new variables $\varepsilon' = \dfrac{\varepsilon_1 + \varepsilon_2}{2}$, $\omega = \varepsilon_1 - \varepsilon_2$; $d\varepsilon_1 d\varepsilon_2 = d\varepsilon' d\omega$

$\Rightarrow \varepsilon_1 = \varepsilon' + \dfrac{\omega}{2}$, $\varepsilon_2 = \varepsilon' - \dfrac{\omega}{2}$

$\Rightarrow$

$$\delta G = \int d\varepsilon' \int d\omega\, \delta(\varepsilon - \varepsilon')\, \delta(\omega - \Omega)\, e^{-i\omega T}\, \frac{1}{i\Omega}\, G(\varepsilon_1)\, \bar{j}\bar{E}_0\, G(\varepsilon_2) =$$

$$= G^{(0)}\left(\varepsilon + \frac{\Omega}{2}\right) \bar{j} \cdot \bar{A}(T)\, G^{(0)}\left(\varepsilon - \frac{\Omega}{2}\right) \equiv \delta G(\varepsilon, T)$$

$$\begin{pmatrix} \delta G^R & \delta G^K \\ 0 & \delta G^A \end{pmatrix} = \begin{pmatrix} G^R_{\varepsilon+\frac{\Omega}{2}} & G^K_{\varepsilon+\frac{\Omega}{2}} \\ 0 & G^A_{\varepsilon+\frac{\Omega}{2}} \end{pmatrix} \begin{pmatrix} jA(T) & 0 \\ 0 & jA(T) \end{pmatrix} \begin{pmatrix} G^R_{\varepsilon-\frac{\Omega}{2}} & G^K_{\varepsilon-\frac{\Omega}{2}} \\ 0 & G^A_{\varepsilon-\frac{\Omega}{2}} \end{pmatrix}$$

for individual components we obtain!

$$\begin{cases} \delta G^R = G^R_{\varepsilon+\frac{\Omega}{2}}\, jA(T)\, G^R_{\varepsilon-\frac{\Omega}{2}} = \frac{1}{i\Omega} e^{-i\Omega T}\, G^R_{\varepsilon+\frac{\Omega}{2}}\, \vec{j}\cdot\vec{E}_0\, G^R_{\varepsilon-\frac{\Omega}{2}} \\ \delta G^A = \frac{1}{i\Omega} e^{-i\Omega T}\, G^A_{\varepsilon+\frac{\Omega}{2}}\, \vec{j}\cdot\vec{E}_0\, G^A_{\varepsilon-\frac{\Omega}{2}} \\ \delta G^K = G^R_{\varepsilon+\frac{\Omega}{2}}\, \vec{j}\vec{A}(T)\, \underbrace{G^K_{\varepsilon-\frac{\Omega}{2}}}_{\substack{\parallel \\ h_{\varepsilon-\frac{\Omega}{2}}\left(G^R_{\varepsilon-\frac{\Omega}{2}} - G^A_{\varepsilon-\frac{\Omega}{2}}\right) \\ \text{FDT}}} + \underbrace{G^K_{\varepsilon+\frac{\Omega}{2}}}_{\substack{\Longrightarrow\ h_{\varepsilon+\frac{\Omega}{2}}\left(G^R_{\varepsilon+\frac{\Omega}{2}} - G^A_{\varepsilon+\frac{\Omega}{2}}\right) \\ \text{FDT}}}\, jA(T)\, G^A_{\varepsilon-\frac{\Omega}{2}} \end{cases}$$

$$\boxed{h_\varepsilon = \text{th}\,\frac{\varepsilon}{2T}}$$

Let us look at the <u>dc</u> response (constant $\vec{E}$ field)

$\Omega \to 0$! $\Rightarrow$ $e^{-i\Omega T} \to 1$

$$\delta G^R \cong \frac{1}{i\Omega} G^R_\varepsilon\, \vec{j}\vec{E}\, G^R_\varepsilon + \frac{1}{2i}\left(\frac{\partial G^R}{\partial \varepsilon}\, jE\, G^R - G^R\, \vec{j}\vec{E}\, \frac{\partial G^R}{\partial \varepsilon}\right)$$

$$\delta G^A \cong \frac{1}{i\Omega} G^A_\varepsilon\, \vec{j}\vec{E}\, G^A_\varepsilon + \frac{1}{2i}\left(\frac{\partial G^A}{\partial \varepsilon}\, \vec{j}\vec{E}\, G^A - G^A\, \vec{j}\vec{E}\, \frac{\partial G^A}{\partial \varepsilon}\right)$$

$$\delta G^K = \underbrace{G^R_{\varepsilon+\frac{\Omega}{2}}\, \overline{jA}\, G^R_{\varepsilon-\frac{\Omega}{2}}}_{\delta G^R} h_{\varepsilon-\frac{\Omega}{2}} - \underbrace{G^A_{\varepsilon+\frac{\Omega}{2}}\, \overline{jA}\, G^A_{\varepsilon-\frac{\Omega}{2}}}_{\delta G^A} h_{\varepsilon+\frac{\Omega}{2}} -$$

$$- \underbrace{G^R_{\varepsilon+\frac{\Omega}{2}}\, \overline{jA}\, G^A_{\varepsilon-\frac{\Omega}{2}} h_{\varepsilon-\frac{\Omega}{2}} + G^R_{\varepsilon+\frac{\Omega}{2}}\, \overline{jA}\, G^A_{\varepsilon-\frac{\Omega}{2}} h_{\varepsilon+\frac{\Omega}{2}}}_{\parallel}$$

$$\left(+ \left(G^R_{\varepsilon+\frac{\Omega}{2}} \frac{jE}{i\Omega} G^A_{\varepsilon-\frac{\Omega}{2}} \right) \left(\underbrace{h_{\varepsilon+\frac{\Omega}{2}} - h_{\varepsilon-\frac{\Omega}{2}}}_{\sim\Omega} \right) \right)$$

Also note that $f(\varepsilon) = \frac{1-h_\varepsilon}{2}$ - is the Fermi distribution function

$$h_\varepsilon = \operatorname{th}\frac{\varepsilon-\mu}{2T}, \quad f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1},$$

$$\Rightarrow \delta G^< = \frac{1}{2}\left(\delta G^K - \delta G^R + \delta G^A\right) = \delta G^R \underbrace{\frac{h_{\varepsilon-\frac{\Omega}{2}}-1}{2}}_{\substack{\parallel \\ -f(\varepsilon-\frac{\Omega}{2})}} - \delta G^A \underbrace{\frac{h_{\varepsilon+\frac{\Omega}{2}}-1}{2}}_{\substack{\parallel \\ -f(\varepsilon+\frac{\Omega}{2})}} + G^R_\varepsilon jE\, G^A_\varepsilon \underbrace{\frac{h_+ - h_-}{2i\Omega}}_{\substack{\parallel \\ \frac{1}{i}\left(-\frac{\partial f}{\partial \varepsilon}\right)}}$$

Thus we eventually obtain:

$$\delta G^< = \underbrace{-\frac{1}{i\Omega}\left(G^R_\varepsilon jE\, G^R_\varepsilon - G^A jE\, G^A\right) f(\varepsilon)}_{\text{diverging part in the limit } \Omega\to 0} + \frac{1}{2i}\left(G^R jE\, G^R + G^A jE\, G^A - 2 G^R jE\, G^A\right)\left(-\frac{\partial f}{\partial \varepsilon}\right)$$

$$- \frac{1}{2i}\left(\frac{\partial G^R}{\partial \varepsilon} jE\, G^R - G^R jE \frac{\partial G^R}{\partial \varepsilon} - \frac{\partial G^A}{\partial \varepsilon} jE\, G^A + G^A jE \frac{\partial G^A}{\partial \varepsilon}\right) f(\varepsilon)$$

$$\delta G^< = -\frac{1}{i\Omega}\left(G^R jE\, G^R - G^A jE\, G^A\right) f + \frac{1}{2i}\left(2G^R jE\, G^A - G^R jE\, G^R - G^A jE\, G^A\right)\left(-\frac{\partial f}{\partial \varepsilon}\right)$$

$$+ \frac{1}{2i}\left(G^R jE \frac{\partial G^R}{\partial \varepsilon} - \frac{\partial G^R}{\partial \varepsilon} jE\, G^R - G^A jE \frac{\partial G^A}{\partial \varepsilon} + \frac{\partial G^A}{\partial \varepsilon} jE\, G^A\right) f(\varepsilon)$$

this term is kind of a problem! It is, however, exactly cancelled by the diamagnetic contribution: $\sim \mathrm{Tr}\, \underline{j\, G^{(0)<}}$

Kubo formula: (we defined δG with a wrong sign, perturbation is $-jA$)

$$\langle j \rangle = i\,\mathrm{Tr}\, j\,\delta G^< = \frac{1}{2}\int \frac{d\varepsilon}{2\pi}\left(-\frac{\partial f}{\partial \varepsilon}\right) \underset{\text{remaining trace}}{\mathrm{Tr}_r} \underbrace{\left[\vec{j}(G^R - G^A)\, \vec{j}\cdot\vec{E}\, G^A - \vec{j}\, G^R\, \vec{j}\cdot\vec{E}\,(G^R - G^A)\right]}_{\text{standard part}}$$

over space

$$+ \frac{1}{2}\int \frac{d\varepsilon}{2\pi} f(\varepsilon)\, \underset{\text{devided by volume}}{\mathrm{Tr}_r} \underbrace{\vec{j}\left[G^R jE \frac{\partial G^R}{\partial \varepsilon} - \frac{\partial G^R}{\partial \varepsilon} jE\, G^R - G^A jE \frac{\partial G^A}{\partial \varepsilon} + \frac{\partial G^A}{\partial \varepsilon} jE\, G^A\right]}_{\text{topological contribution}}$$

In two dimensions:

$$\vec{j} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

There is no topological contribution to σ_{xx}

$$\sigma_{xx} = \frac{1}{2}\int \frac{d\varepsilon}{2\pi}\left(-\frac{\partial f}{\partial \varepsilon}\right)\frac{\mathrm{Tr}}{V}\left(j_x (G^R - G^A) j_x G^A - j_x G^R j_x (G^R - G^A)\right)$$

but it is different for $\sigma_{xy} = \sigma_{xy}^{I} + \sigma_{xy}^{II}$

$$\sigma_{xy}^{I} = \frac{1}{2}\int \frac{d\varepsilon}{2\pi}\underbrace{\left(-\frac{\partial f}{\partial \varepsilon}\right)}_{\text{pinned to Fermi-surface}}\frac{\mathrm{Tr}}{V}\left(j_x (G^R - G^A) j_y G^A - j_x G^R j_y (G^R - G^A)\right)$$

$$\sigma_{xy}^{II} = \frac{1}{2}\int \frac{d\varepsilon}{2\pi} f(\varepsilon) \frac{\mathrm{Tr}}{V}\left(j_x G^R j_y \frac{\partial G^R}{\partial \varepsilon} - j_x \frac{\partial G^R}{\partial \varepsilon} j_y G^R - j_x G^A j_y \frac{\partial G^A}{\partial \varepsilon} + j_x \frac{\partial G^A}{\partial \varepsilon} j_y G^A\right)$$

depends on Fermi-see $\longrightarrow$ responsible for QHE

in p-space we normally have $G_p^R = \dfrac{1}{\varepsilon - \varepsilon_p + i\gamma}$

$\frac{1}{V}\mathrm{Tr}_r \rightarrow \int \frac{d^2 p}{(2\pi)^2}$ in p-representation

volume

$\frac{\partial G_p^R}{\partial \varepsilon} = - G_p^R G_p^R$ — Ward identity

we needed $G^<(r,r)$
if $G^<(r,r)$ does not depend on r (translational invariance)
we also can compute:

$\frac{1}{V}\underbrace{\int dr\, G^<(r,r)}_{\mathrm{Tr}_r G^<} = G^<(r,r)$

$\Rightarrow G^<(r,r) = \frac{1}{V}\mathrm{Tr}_r G^<$

Some words on the topological contribution to the conductivity

$$\sigma_{xy}^{II} = \frac{1}{2}\frac{e^2}{2\pi}\int d\varepsilon\, f(\varepsilon)\, \mathrm{Tr}\left(v_x G^R v_y \frac{\partial G^R}{\partial \varepsilon} - v_x \frac{\partial G^R}{\partial \varepsilon} v_y G^R - v_x G^A v_y \frac{\partial G^A}{\partial \varepsilon} + v_x \frac{\partial G^A}{\partial \varepsilon} v_y G^A\right)$$

I have used $\hat{j} = e\hat{v}$ $\boxed{\vec{v} = \nabla_p H}$ ← velocity operator

thinking more about σ_{xy}^{II} remember $[\hat{x},\hat{p}] = i$ - in quantum mechanics

in momentum representation $\vec{r} = i\vec{\nabla}_p$ (same as $\vec{p} = -i\vec{\nabla}_r$ in coordinate one)

$\Rightarrow$ $G_p^{-1} = \varepsilon - H_p + i\gamma$, $H_p = \frac{p^2}{2m}$ for example

$\Rightarrow$ $\underbrace{[r, G^{-1}] = -iv}$ or $\underbrace{i(G\vec{r} - \vec{r}G) = G\vec{v}G}$

we can now use $\frac{\partial G^R}{\partial \varepsilon} = -(G^R)^2$ and write:

$$\mathrm{Tr}\left(v_x G v_y \frac{\partial G}{\partial \varepsilon} - v_x \frac{\partial G}{\partial \varepsilon} v_y G\right) = \mathrm{Tr}\left(v_x \underbrace{G^2 v_y G} - v_x G v_y G^2\right) =$$

$$= \mathrm{Tr}\left[v_x G \cdot i(Gy - yG) - i(Gx - xG) v_y G\right] =$$

$$\boxed{=}\; i\,\mathrm{Tr}\left(\underline{y\, v_x G^2} - v_x G y G \underline{- x\, v_y G^2} + v_y G x G\right)$$

$$= \underbrace{i\,\mathrm{Tr}\left[(y v_x - x v_y)\, G^2\right]}_{\parallel\!\!\;\parallel} - i\,\mathrm{Tr}\Big(\cancel{v_x}\, \underbrace{i y\,(Gx - xG)}_{=0} \underbrace{- i x\,(Gy - yG)}_{=0}\Big)$$

using cyclic permutations under trace! and the fact that $\underline{xy = yx}$

$$\underline{-i\,\mathrm{Tr}\,\frac{\partial G}{\partial \varepsilon}\,(y v_x - x v_y)}$$ ← applies to both $G = G^R$ & $G = G^A$!

$$\Rightarrow \quad \sigma^{II}_{xy} = \frac{1}{2}\,\frac{e^2}{2\pi}\int d\varepsilon\; f(\varepsilon)\,(-i)\,\mathrm{Tr}\,\frac{\partial(G^R - G^A)}{\partial \varepsilon}\,(v_x y - v_y x) =$$

$$= \frac{1}{2i}\,\frac{e^2}{2\pi}\int d\varepsilon\left(-\frac{\partial f}{\partial \varepsilon}\right)\mathrm{Tr}\Big[(G^R - G^A)\,\underbrace{(v_x y - v_y x)}\Big]$$

by parts

pinned to the Fermi level

non-zero only at the sample boundaries (edge states)

Defines QHE

Further simplifications of σ_{xy}^{II} (Bulk-edge correspondence)

Consider $\frac{\partial G^R}{\partial B} = \frac{\partial G^R}{\partial \varepsilon} \cdot \left(-\frac{\partial H_p}{\partial B}\right)$ use $\vec{A} = \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \frac{B}{2}$

magnetic field

$$H_p = H_p\left(p_x + ey\frac{B}{2},\ p_y - ex\frac{B}{2}\right)$$

$$G^R = \frac{1}{\varepsilon - H_p + i\gamma}$$

$$\Rightarrow \frac{\partial H_p}{\partial B} = \frac{ey}{2}\frac{\partial H_p}{\partial p_x} - \frac{ex}{2}\frac{\partial H_p}{\partial p_y} = \underline{\frac{e}{2}(y v_x - x v_y)}$$

$$\Rightarrow \frac{\partial G^R}{\partial B} = -\frac{e}{2}\frac{\partial G^R}{\partial \varepsilon}(v_x y - v_y x)$$ and the same for G^A

$$\Rightarrow \sigma_{xy}^{II} = \frac{e}{2\pi} i \int d\varepsilon\, f(\varepsilon)\, \mathrm{Tr}\, \frac{\partial (G^R - G^A)}{\partial B} = \underline{-} e \frac{\partial}{\partial B} \underbrace{\int d\varepsilon\, f(\varepsilon) \overbrace{\mathrm{Im}\, G^R \frac{1}{\pi}}^{\text{density of states}}}_{\equiv\ n\ -\ \text{electron concentration}}$$

$$= \underline{\frac{\partial}{\partial B}(en)}$$ $\leftarrow$ counts the number of Landau levels under the Fermi surface

Bulk representation

$$n = \nu \frac{eB}{2\pi}$$

$$\boxed{\sigma_{xy}^{II} = \frac{e^2}{2\pi}\nu = \frac{e^2}{h}\nu}$$ $\leftarrow$ chern number

$$n = \int d\varepsilon\, \nu(\varepsilon) f(\varepsilon)$$

$$\nu(\varepsilon) = -\frac{1}{\pi}\, \mathrm{Im}\, G^R$$

Degeneracy of Landau Level

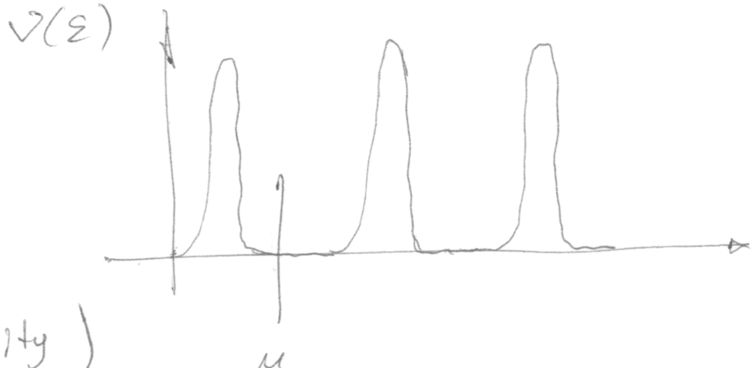
$$N_\varphi = \frac{\Phi}{\varphi_0} = \frac{B\mathcal{A}}{h/e} = \frac{eB\mathcal{A}}{2\pi}$$

area

filling factor

$$n = \nu \frac{N_\varphi}{\mathcal{A}} = \nu \frac{eB}{2\pi}$$

In the case of quantum-Hall effect, the spectrum consists of Landau-levels (broadened by disorder). Therefore, the density of states is very non-monotonous:



← The degeneracy N_Φ of each level is the ratio of total magnetic flux through the sample $\Phi = BA$ to the elementary flux quantum $\Phi_0 = h/e$

$$N_\Phi = \frac{\Phi}{\Phi_0} = \frac{eBA}{h} \quad (\text{in our units } h = 2\pi\hbar = 2\pi)$$

The electron concentration is, therefore,

$$n = \nu \frac{N_\Phi}{A}, \quad \text{where } \nu \text{ is the filling factor}$$

$$n = \nu \frac{eB}{h} \Rightarrow \text{from Středa formula}$$

we, therefore, have

$$\sigma_{xy}^{II} = \frac{\partial}{\partial B}(en) = \nu \frac{e^2}{h} \quad \leftarrow \text{very general result.}$$

(mobility gap)

for μ (chemical potential) laying in between Landau levels we have $\nu \to 0$ + localization!

$$\Rightarrow \sigma_{xx} = \sigma_{xy}^{I} = 0 \quad (\text{with exponential precision})$$

In this regime what remains is only topological contribution:

$$\sigma_{xy} = \sigma_{xy}^{II} = \nu \frac{e^2}{h}, \quad \rho_{xy} = \frac{1}{\nu}\frac{h}{e^2} \quad \leftarrow \text{integer quantum Hall effect}$$

Other response functions

Consider, for example, response of electron spin (spin polarization) to electric field applied:

$$\vec{S} = \frac{1}{2}(-i)\,\mathrm{Tr}(\vec{\sigma}\,\delta G^<) = -\frac{i}{2}\,\mathrm{Tr}\,\vec{\sigma}\,\delta G^<(r,t;r,t)$$

trace

trace in spin space

coinciding arguments

defines effective magnetization as a response to electric field (or electric current)

Such response is trivial except of the systems with spin-orbit coupling; e.g.

$$H = \frac{p^2}{2m} + \alpha\underbrace{(\bar{\sigma}\times\bar{p})_z}_{\sigma_x p_y - \sigma_y p_x} + V(r) \quad \leftarrow \text{2D Rashba model}$$

$$p \to p - eA$$

Current operator in this case: $\hat{j} = e\frac{\partial H}{\partial p} = -\frac{\delta H}{\delta A}$

$$j_{r,x} = \frac{-ie}{2m}(\vec{\nabla} - \overleftarrow{\nabla}) - e\sigma_y$$

$$j_{r,y} = \frac{-ie}{2m}(\vec{\nabla} - \overleftarrow{\nabla}) + e\sigma_x$$

in momentum space:

$$j_{p,x} = e\left(\frac{p}{m} - \sigma_y\right)$$

$$j_{p,y} = e\left(\frac{p}{m} + \sigma_x\right)$$

The same derivation as above gives

$$\vec{S} = \hat{K}\vec{E}$$

$\hat{K}$ - is a 6-component tensor
since $\vec{S} = (S_x, S_y, S_z)$ - three projections
and $\vec{E} = (E_x, E_y)$ - in-plane field

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in space; convolution in space

$$K^{I}_{\alpha\beta} = \frac{1}{2}\int\frac{d\varepsilon}{2\pi}\left(-\frac{\partial f}{\partial\varepsilon}\right)\frac{\mathrm{Tr}}{V}\left(\frac{1}{2}\sigma_\alpha\,(G^R-G^A)\,j_\beta\,G^A - \frac{1}{2}\sigma_\alpha\,G^R\,j_\beta\,(G^R-G^A)\right)_{\text{av}}$$

~~in space~~ ~~at coinciding arguments~~ (assumed translational invariance on average

$$K^{II}_{\alpha\beta} = \frac{1}{2}\int\frac{d\varepsilon}{2\pi}\,f(\varepsilon)\,\frac{1}{V}\mathrm{Tr}\left(\frac{1}{2}\sigma_\alpha\,G^R j_\beta\frac{\partial G^R}{\partial\varepsilon} - \frac{1}{2}\sigma_\alpha\frac{\partial G^R}{\partial\varepsilon}j_\beta\,G^R - \frac{1}{2}\sigma_\alpha\,G^A j_\beta\frac{\partial G^A}{\partial\varepsilon} + \frac{1}{2}\sigma_\alpha\frac{\partial G^A}{\partial\varepsilon}j_\beta\,G^A\right)$$

Can you analyze that assuming?

(you need more than that for disorder averaging, namely, vertex corrections!!!)

$$\hat{G}^R_p = \frac{1}{\varepsilon - \hat{h}_p + i\gamma} \qquad \hat{h}_p = \frac{p^2}{2m} + \alpha(\vec{\sigma}\times\vec{p})_z$$

$$\hat{G}^A_p = \frac{1}{\varepsilon - \hat{h}_p - i\gamma}$$

convolution in $\underline{r}$ => to simple product in $\underline{p}$?

since G^R, G^A are equilibrium functions

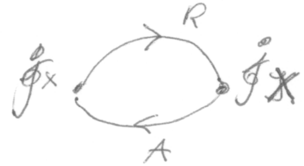
Remarks on disorder averaging

You have to compute $\langle \sigma_{xx} \rangle$, for example, where brackets now mean the additional averaging over the ensemble of disordered systems.

Let us consider how it is done for the main contribution to σ_{xx}

at zero temperature

$$\langle \sigma_{xx} \rangle = \frac{1}{2\pi} \left\langle \frac{1}{V} \mathrm{Tr} \left(j_x G^R j_x G^A \right) \right\rangle =$$



averaging procedure is separated into the averaging of Green's functions individually:

$$\sigma_{xx}^{bare} = \frac{1}{2\pi} \frac{1}{V} \mathrm{Tr} \left(j_x \langle G^R \rangle j_x \langle G^A \rangle \right) \quad ; \quad \langle G^R \rangle =$$

disorder potential

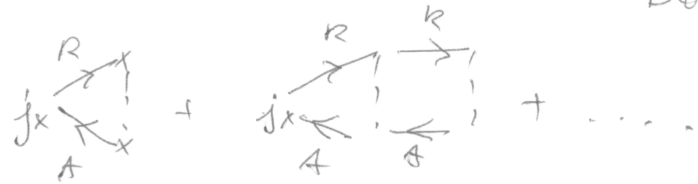
Born approximation

and the calculation of vertex corrections:

self-consistent Born approximation

$$j_x \to \Gamma_x$$

$$\Gamma_x = j_x +$$



eventually $\sigma_{xx} = \frac{1}{2\pi} \frac{1}{V} \mathrm{Tr} \left(\Gamma_x \langle G^R \rangle j_x \langle G^A \rangle \right)$